Montague Grammar

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Overview

- Introduction
- Type Theory
- A Montague-Style Grammar
- Scope Ambiguities
- Summary

Introduction

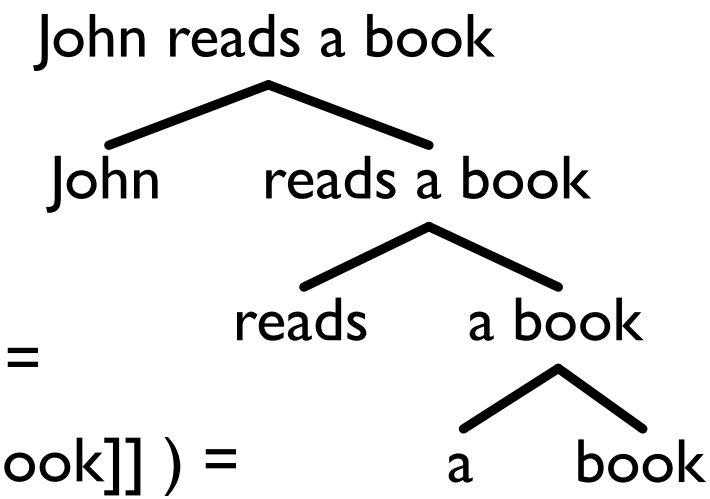
- The basic assumption underlying Montague Grammar is that the meaning of a sentence is given by its truth conditions.
  - "Peter reads a book" is true iff Peter reads a book
- Truth conditions can be represented by logical formulae
  - "Peter reads a book" → $\exists x(\text{book}(x) \wedge \text{read}(p^*, x))$
- Indirect interpretation:
  - natural language → logic → models

Compositionality

- An important principle underlying Montague Grammar is the so called "principle of compositionality"

  The meaning of a complex expression is a function of the meanings of its parts, and the syntactic rules by which they are combined (Partee & al, 1993)

Compositionality



[[John reads a book]] =

C_1([[John]], [[reads a book]]) =

C_1([[John]], C_2([[reads]] , [[a book]]) =

C_1([[John]], C_2([[reads]] , C_3([[a]], [[book]]))

Representing Meaning

- First order logic is in general not an adequate formalism to model the meaning of natural language expressions.
- Expressiveness
  - "John is an intelligent student" ⇒ intelligent(j*) ∧ stud(j*)
  - "John is a good student" ⇒ good(j*) ∧ stud(j*) ??
  - "John is a former student" ⇒ former(j*) ∧ stud(j*) ???
- Representations of noun phrases, verb phrases, …
  - "is intelligent" ⇒ intelligent(·) ?
  - "every student" ⇒ ∀x(student(x) ⇒ ·) ???

Type Theory

- First order logic provides only n-ary first order relations, which is insufficient to model natural language semantics.
- Type theory is more expressive and flexible – it provides higher-order relations and functions of different kinds.
- Some type theoretical expressions
  - "John is a good student" ⇒ good(student)(j*)
  - "is intelligent" ⇒ intelligent
  - "every student" ⇒ λP∀x(student(x) ⇒ P(x))

Types

- A set of basic types, for instance {e, t}
  - e is the type of individual terms ("entity")
  - t is the type of formulas ("truth value")
- The set T of types is the smallest set such that
  - if σ is a basic type, then σ is a type
  - if σ, τ are types, then $\langle \sigma, \tau \rangle$ is a type
- The type $\langle \sigma, \tau \rangle$ is the type of functions that map arguments of type σ to values of type τ.

Some Example Types

- One-place predicate constant: sleep, walk, student, …
  - ‹e, t›
- Two-place relation: read, write, …
  - ‹e, ‹e,t››
- Attributive adjective: good, intelligent, former, …
  - ‹‹e,t›, ‹e,t››

Vocabulary

- Pairwise disjoint, possibly empty sets of non-logical constants:
  - Con_τ, for every type τ
- Infinite and pairwise disjoint sets of variables:
  - Var_τ, for every type τ
- Logical constants:
  - $\forall, \exists, \wedge, \neg, \ldots, \lambda$

Syntax

- For every type τ, we define the set of meaningful expressions ME_τ as follows:
  - $Con_\tau \subseteq ME_\tau$ and $Var_\tau \subseteq ME_\tau$, for every type τ
  - If $\alpha \in ME_{\langle\sigma,\tau\rangle}$ and $\beta \in ME_\sigma$, then $\alpha(\beta) \in ME_\tau$.
  - If $A, B \in ME_t$, then so are $\neg A$, $(A \wedge B)$, $(A \Rightarrow B)$, …
  - If $A \in ME_t$, then so are $\forall x A$ and $\exists x A$, where x is a variable of arbitrary type.
  - If α, β are well-formed expressions of the same type, then $\alpha = \beta \in ME_t$.
  - If $\alpha \in ME_\tau$ and $x \in Var_\sigma$, then $\lambda x \alpha \in ME_{\langle\sigma,\tau\rangle}$.

Some Examples

- "John works."

$$\frac{j^* \in ME_e \qquad work \in ME_{\langle e,\, t\rangle}}{work(j^*)}$$

- "Every student works."

$$\frac{\dfrac{every \in ME_{\langle\langle e,\, t\rangle,\, \langle\langle e,\, t\rangle,\, t\rangle} \qquad student \in ME_{\langle e,\, t\rangle}}{every(student) \in ME_{\langle\langle e,\, t\rangle,\, \langle\langle e,\, t\rangle,\, t\rangle} \qquad work \in ME_{\langle e,\, t\rangle}}}{every(student)(work) \in ME_t}$$

Semantics

- Let U be a non-empty set of entities. For every type τ, the domain of possible denotations D_τ is given by
  - $D_e = U$
  - $D_t = \{0, 1\}$
  - $D_{\langle\sigma, \tau\rangle}$ = the set of functions from D_σ to D_τ
- A model structure is a structure $M = (U_M, V_M)$
  - U_M is a non-empty set of individuals
  - V_M is a function that assigns every non-logical constant of type τ an element of D_τ.
- Variable assignment g: $Var_\tau \rightarrow D_\tau$

Semantics

- Let M be a model structure and g a variable assignment
  - $[[\alpha]]^{M,g} = V_M(\alpha)$, if α is a constant
  - $[[\alpha]]^{M,g} = g(\alpha)$, if α is a variable
  - $[[\alpha(\beta)]]^{M,g} = [[\alpha]]^{M,g}([[\beta]]^{M,g})$
  - $[[\neg\varphi]]^{M,g} = 1$ iff $[[\varphi]]^{M,g} = 0$
  - $[[\varphi \wedge \psi]]^{M,g} = 1$ iff $[[\varphi]]^{M,g} = 1$ and $[[\psi]]^{M,g} = 1$, etc.
  - $[[\exists v\varphi]]^{M,g} = 1$ iff there is a $\in D_\tau$ such that $[[\varphi]]^{M,g[v/a]} = 1$
  - $[[\forall v\varphi]]^{M,g} = 1$ iff for all a $\in D_\tau$, $[[\varphi]]^{M,g[v/a]} = 1$
  - $[[\alpha = \beta]]^{M,g} = 1$ iff $[[\alpha]]^{M,g} = [[\beta]]^{M,g}$

Semantics of λ-Expressions

- Let M be a model structure and g a variable assignment
  - If $\alpha \in ME_\tau$ and $v \in Var_\sigma$, then $[[\lambda v \alpha]]^{M,g}$ is that function f from D_σ to D_τ such that for any $a \in D_\sigma$, $f(a) = [[\alpha]]^{M,g[v/a]}$
- "Syntactic shortcut:" β-reduction
  - $(\lambda x \varphi)(\psi) \equiv \varphi[\psi/x]$

    if all free variables in ψ are free for x in φ
  - A variable y is free for x in φ if no free occurence of x in ψ is in the scope of a ∃y, ∀y, λy

Noun Phrases

- "John works" → work(j*)
- "A student works." → $\exists x(\text{student}(x) \wedge \text{work}(x))$
- "Every student works." → $\forall x(\text{student}(x) \Rightarrow \text{work}(x))$
- "John and Mary work." → work(j*) ∧ work(m*)

Noun Phrases

- Using λ-abstraction, noun phrases can be given a uniform interpretation as "generalized quantifiers"
  - "John" → $\lambda P.P(j^*)$
  - "A student" → $\lambda P \exists x(student(x) \wedge P(x))$
  - "Every student" → $\lambda P \forall x(student(x) \Rightarrow P(x))$
  - "John and Mary" → $\lambda P.P(j^*) \wedge P(m^*)$

Noun Phrases

- "John works"

$$\frac{\lambda P.P(j^*) \in ME_{\langle\langle e,t\rangle,t\rangle} \quad work \in ME_{\langle e,t\rangle}}{\frac{(\lambda P.P(j^*))(work) \in ME_t}{work(j^*) \in ME_t}}$$

- "Every student works."

$$\frac{\lambda P \forall x(student(x) \Rightarrow P(x)) \in ME_{\langle\langle e,t\rangle,t\rangle} \quad work \in ME_{\langle e,t\rangle}}{\frac{(\lambda P \forall x(student(x) \Rightarrow P(x)))(work) \in ME_t}{\forall x(student(x) \Rightarrow work(x)) \in ME_t}}$$

Determiners

- Determiners like "a," "every," "no" denote higher order functions taking (denotations of) common nouns and return a higher order relation.
  - "every" → $\lambda P \lambda Q \forall x(P(x) \Rightarrow Q(x))$
  - "some" → $\lambda P \lambda Q \exists x(P(x) \wedge Q(x))$
  - "no" → $\lambda P \lambda Q \neg \exists x(P(x) \wedge Q(x))$
- "Every student"

$$\dfrac{\dfrac{\lambda P \lambda Q \forall x(P(x) \Rightarrow Q(x)) \qquad \text{student}}{(\lambda P \lambda Q \forall x(P(x) \Rightarrow Q(x)))(\text{student})}}{\lambda Q \forall x(\text{student}(x) \Rightarrow Q(x))}$$

A Montague-Style Grammar for a Fragment of English

Syntactic Component

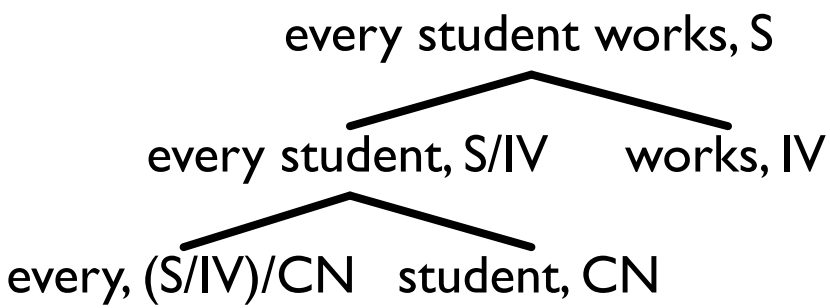
- Montague Grammar is based upon (a particular version of) categorial grammar.
- The set of categories is the smallest set such that
  - S, IV, CN are categories
  - If A, B are categories, then A/B is a category
- Some categories
  - IV/T [= TV] transitive verbs
  - S/IV [= T] terms (= noun phrases)
  - T/CN determiners

Lexicon

- For each category A, we assume a possibly empty set B_A of basic expressions of category A.
- For instance
  - B_T = { John, Mary, he_0, he_1, … }
  - B_{CN} = { student, man, woman, … }
  - B_{IV} = { sleep, work, … }
  - $B_{IV/T}$ = { read, … }
  - $B_{T/CN}$ = { a, every, no, the, … }

Syntactic Rules (Simplified)

- General rule schema:
  - $B_A \subseteq P_A$
  - If $\alpha \in P_A$ and $\delta \in P_{B/A}$, then $\delta\alpha \in P_B$
- "Every student works"



Translation into Type Theory

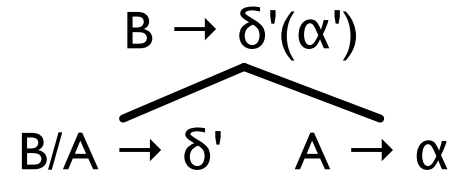
- A translation of natural language into type theory is a homomorphism that assigns each $\alpha \in P_A$ an $\alpha' \in ME_{f(A)}$
- f maps categories to types as follows
  - f(S) = t
  - f(CN) = f(IV) = ‹e, t›
  - f(A/B) = ‹f(B), f(A)›

Translation: Lexical Categories

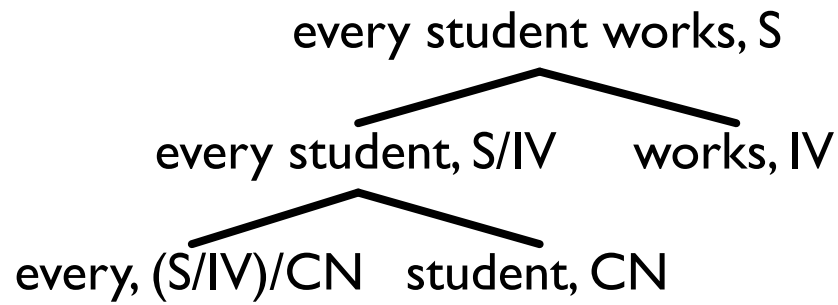
- “John” → $\lambda P.P(j^*)$
- “every” → $\lambda P \lambda Q \forall x(P(x) \Rightarrow Q(x))$
- “a” → $\lambda P \lambda Q \exists x(P(x) \wedge Q(x))$
- “student” → student
- “book” → book
- “works” → work
- …

Translation: Phrasal Categories

- Syntactic rule:
  - If $\alpha \in P_A$ and $\delta \in P_{B/A}$, then $\delta\alpha \in P_B$
- Corresponding translation rule:
  - If $\alpha \rightarrow \alpha'$, $\delta \rightarrow \delta'$, then $\delta\alpha \rightarrow \delta'(\alpha')$



“Every student works”



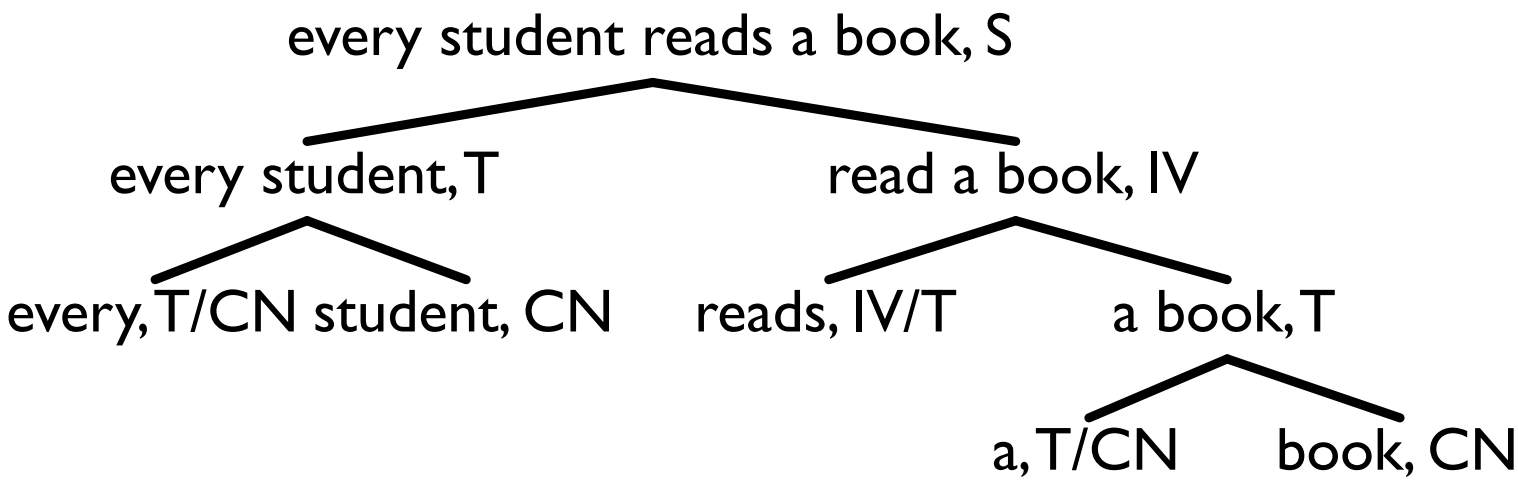
- “every” → λPλQ∀x(P(x) ⇒ Q(x))
- “student” → student
- “every student” → λPλQ∀x(P(x) ⇒ Q(x))(student)
  = λQ∀x(student(x) ⇒ Q(x))
- “every student works” → λQ∀x(student(x) ⇒ Q(x))(work)
  = ∀x(student(x) ⇒ work(x))

Transitive Verbs

- Transitive verbs have category IV/T (= IV/(S/IV)), the corresponding type is ‹‹e, t›, t›, ‹e, t››
- On the other hand, transitive verbs like "read," "present," … denote a two-place first order relation (type ‹e, ‹e, t››)
  - "John reads a book" → $\exists y(book(y) \wedge read(y)(j^*))$

- "read" → $\lambda Q \lambda x.Q(\lambda y.read^*(y)(x))$
  - $read^* \in ME_{\langle e, \langle e, t \rangle \rangle}$

“Every student reads a book”



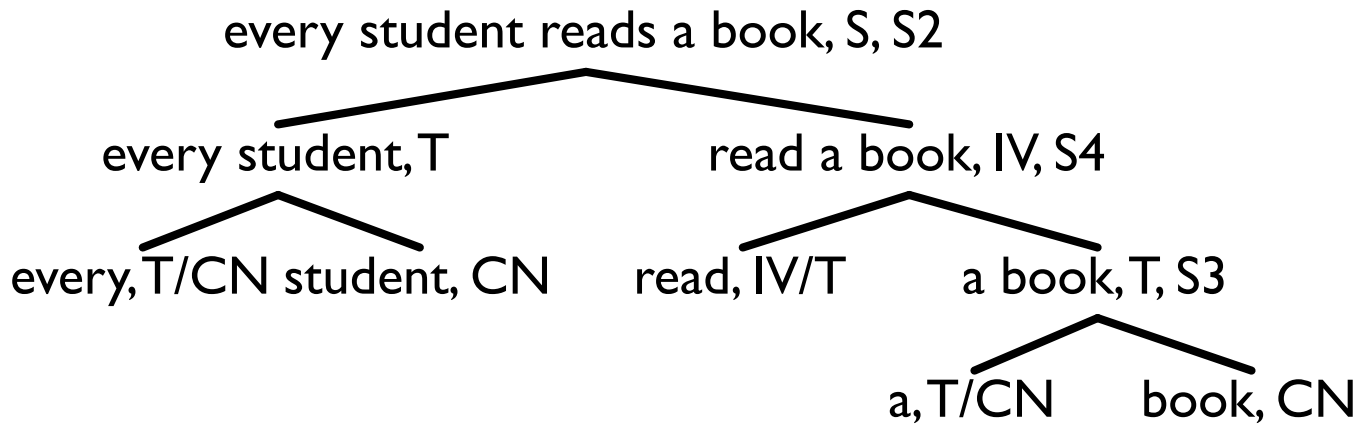
“Every student reads a book”

- “a book” → $\lambda P \exists z(book(z) \wedge P(z))$
- “reads” → $\lambda Q \lambda x.Q(\lambda y.read^*(y)(x))$
- “reads a book”
  - → $\lambda Q \lambda x.Q(\lambda y.read^*(y)(x))(\lambda P \exists z(book(z) \wedge P(z)))$
  - → $\lambda x.\lambda P \exists z(book(z) \wedge P(z))(\lambda y.read^*(y)(x))$
  - → $\lambda x.\exists z(book(z) \wedge (\lambda y.read^*(y)(x))(z))$
  - → $\lambda x.\exists z(book(z) \wedge read^*(z)(x))$
- “every student reads a book”
  - → $\lambda P \forall w(student(w) \Rightarrow P(w))(\lambda x.\exists z(book(z) \wedge read^*(z)(x))$
  - → $\forall w(student(w) \Rightarrow \exists z(book(z) \wedge read^*(z)(w)))$

Scope

- Sentences with multiple scope bearing operators – e.g., quantified noun phrases or negations – are often ambiguous.
- "Every student reads a book"
  - $\forall x(\text{student}(x) \Rightarrow \exists y(\text{book}(y) \wedge \text{read}(y)(x)))$
  - $\exists y(\text{book}(y) \wedge \forall x(\text{student}(x) \Rightarrow \text{read}(y)(x)))$
- "Every student did not pay attention"
  - $\forall x(\text{student}(x) \Rightarrow \neg \text{ pay attention}(x))$
  - $\neg \forall x(\text{student}(x) \Rightarrow \text{pay attention}(x))$

The Problem

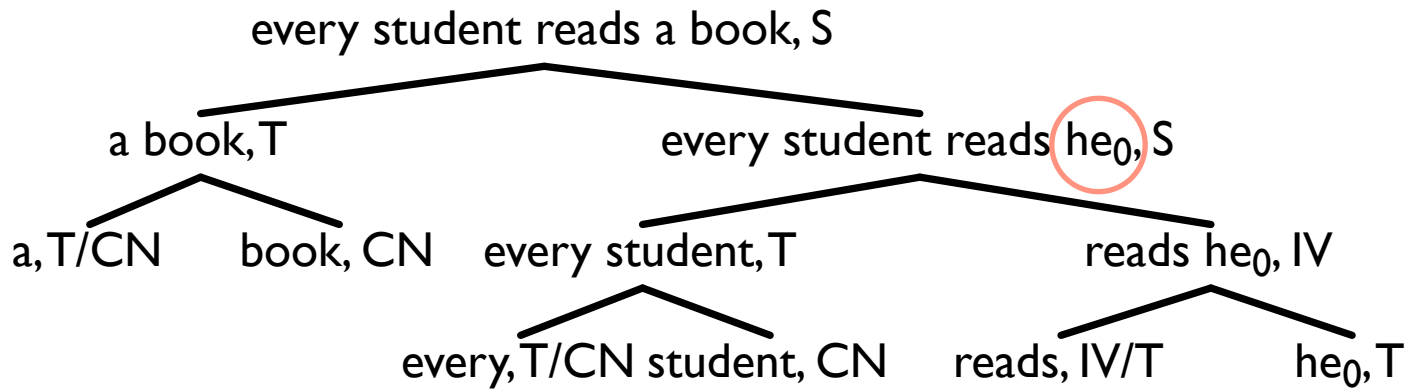


- The principle of compositionality implies that syntactic derivation trees are mapped to a unique type theoretical semantic representation.
- Hence the second reading cannot be derived, unless …

"Montague's Trick"

- Special rule of quantification (aka "Quantifying-in")
  - Terms $\alpha \in P_T$ can combine with sentences $\xi \in P_S$ to form a sentence $\xi' \in P_S$,
  - where ξ' is obtained from ξ by replacing all occurrences of "he_i" with α.
  - For instance: "a book" + "… he_1 …" = "… a book …"
- Sentences can be assigned distinct syntactic derivations

"Montague's Trick"



- "he_0" → $\lambda P.P(x_0)$
- "every student reads he_0" → $\forall y(student(y) \Rightarrow read(x_0)(y))$
- "every student reads a book"
  → $\lambda P \exists x(book(x) \wedge P(x))(\lambda x_0 \forall y(student(y) \Rightarrow read(x_0)(y)))$
  → $\exists x(book(x) \wedge \forall y(student(y) \Rightarrow read(x)(y)))$

“Montague’s Trick”

- The quantification rule allows to derive different scope readings of ambiguous sentences, but …
  - the syntax is made more ambiguous than it actually is
  - no surface oriented analysis

Summary

- The principle of compositionality
  - links syntax and semantics of natural language
- Type theory offers
  - flexibility
  - expressiveness
- Montague like semantics construction …
  - follows the principle of compositionality
  - assumes a strict one-to-one correspondence between syntax and corresponding semantic representations,
  - but needs a "trick" to model scope ambiguities

3.9.19 Montague Grammar Montague grammar is a theory of semantics, and of the relation of semantics to syntax, originally developed by the logician Richard Montague (1930-1971) and subsequently modified and extended by linguists, philosophers, and logicians. Classical Montague grammar had its roots in logic and the philosophy of language; it quickly became influential in linguistics, and linguists have played a large role in its evolution into contemporary formal semantics. â€¢ The basic assumption underlying Montague Grammar is that the meaning of a sentence is given by its truth conditions. - â€œPeter reads a bookâ€ is true iff Peter reads a book. â€¢ Truth conditions can be represented by logical formulae. - â€œPeter reads a bookâ€ â† âˆƒx(book(x) âˆ§ read(p*, x)). â€¢ Indirect interpretation: - natural language â† logic â† models. Compositionality. Montague grammar is an approach to natural language semantics, named after American logician Richard Montague. The Montague grammar is based on formal logic, especially higher-order predicate logic and lambda calculus, and makes use of the notions of intensional logic, via Kripke models. Montague pioneered this approach in the 1960s and early 1970s. Montague's thesis was that natural languages (like English) and formal languages (like programming languages) can be treated in the same way