ELSEVIER Journal of Pure and Applied Algebra 143 (1999) 309–311

JOURNAL OF PURE AND APPLIED ALGEBRA

www.elsevier.com/locate/jpaa

The origins of mathematical abstraction

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Mike Barr, with his students and collaborators, has been a potent influence in the development of category theory. For a small example I recall with gratitude his contribution to a lecture which I gave many years ago at a categorical meeting at McGill. At the time, I was interested in using categories to organize universal algebra. For this I had concocted a device to reduce algebraic systems on several sets to those on a single set. My device was wrong, and Mike was the one to save me from error on this point.

Category theory is a major example of abstraction in mathematics. Abstract structures, applying in many different instances, and described axiomatically, have been a prominent feature of 20th century mathematics. The origin of such abstractions is often formulated in a very simple way: David Hilbert's famous and influential book on the foundations of Euclidean geometry featured the axiomatic method. Hilbert subsequently brought Emmy Noether to Göttingen as his assistant; she founded "modern" algebra by using an axiomatic description of noetherian rings (those with a chain condition); she inspired many students, and then Bourbaki applied the axiomatic method to all mathematics. This triad (Hilbert to Noether to Nicholas), like the famous Baseball triple play (Tinker to Evans to Chance) has been a favorite explanation of the history of abstract 20th century mathematics. But this neat tripartite origin is far too simple. There were many other players and many topics besides algebra involved in effective abstraction. For that matter, there was even some connection with the contemporary development of abstraction in art; I will not try here to document these artistic connections (Flatland to Duchamps and his "Nude descending the staircase"). I will however discuss several of the fields of mathematics which employed the axiomatic method – because this exhibits part of the background for the extensive applications of categories. To display the growth of abstract and axiomatic development, I first present a list of such influential books and papers, beginning with Hilbert, as follows:

PII: S0022-4049(98)00116-9

1898	David Hilbert	Grundlagen der Geometrie
1900	David Hilbert (Paris)	Probleme der Mathematik
1900	Richard Dedekind	Durch Modulen erzeugte Dualgruppe
1903	Bertrand Russell	Principles of Mathematics
1907	Ernest Zermelo	The Axiom of Choice
1910	Whitehead/Russell	Principia Mathematica
1913	Ernest Zermelo	Axiomatic Set Theory
1913	Felix Hausdorff	Mengenlehre (Paul Mongré)
1913	Hermann Weyl	The Idea of the Riemann Surface
1912	E.H. Moore	General Analysis
1921	Emmy Noether	Idealtheorie im Ringbereiche
1929	Otto Haupt	Einführung in die Algebra (2 vols.)
1930	Helmut Hasse	Algebraic Number Theory (Class Fields)
1930	B.L. van der Waerden	Moderne Algebra (2 vols.)
1930	Marston Morse	Calculus of Variations in the Large
1931	Kurt Gödel	Undecideable Propositions
1932	M.H. Stone	Linear Transformations, Hilbert Space
1933	Garrett Birkhoff	On the Combination of Subalgebras
1933	Stefan Banach	Théorie des Operations Lineares
1934	Gerhard Gentzen	Cut Elimination in Proof Theory
1935	Hermann Weyl	Group Theory and Quantum Mechanics
1935	Oystein Ore	On the Foundations of Abstract Algebra
1936	A.M. Turing	On Computable Numbers (Turing Machines)
1936	Alfred Tarski	Truth in Formalized Languages
1936	S.C. Kleene	General Recursive Functions
1937	Solomon Lefschetz	Algebraic Geometry
1938	Seifert/Threllfall	Algebraic Topology
1939	Oscar Zariski	Singularities of Algebraic Surfaces
1939	L.S. Pontragin	Topological Groups (English trans.)
1940	Kurt Gödel	Consistency of the Continuum Hypothesis
1940	John von Neumann	Continuous Geometry; Rings of Operators
1941	Birkhoff/Mac Lane	Survey of Modern Algebra
1942	Nicholas Bourbaki	Eléments de Mathématique
1942	Emil Artin	Galois Theory
1943	Eilenberg/Mac Lane	Homotopy and Homology Groups
1945	Nathan Jacobson	Structure Theory, Simple Rings

This list – or any such list – is necessarily incomplete; there are certainly other items of importance which might well be included. But all these items were really involved in the development of the axiomatic method up till 1945; in particular, they were all known to Eilenberg and Mac Lane at the time of the discovery of category theory – a development which clearly depended upon interaction with various different mathematical topics. It still depends upon such interactions.

The works here listed were relevant not only to category theory, but to a number of other mathematical directions, as follows:

In Algebra:	Abstract algebra
	Galois theory
	Universal algebra
	Lattice theory
	Class field theory
In Analysis:	Functional analysis
	Rings of operators
	Topological groups
In Geometry:	Algebraic Topology
	Algebraic Geometry
In Foundations:	Set theory
	Mathematical logic
	Recursive functions
	Model theory

In this whole context, category theory was by no means alone.

It is striking to note the many aspects of the influence of David Hilbert in many of these developments:

In axiomatics of geometry;
in algebraic number theory, as in his famous report;
in invariant theory;
in integral equations (Hilbert Space; functional analysis);
in logic, in his formulation of proof theory and first order logic.

An abstract is a part or arbitrary partition of something, as well as it is the drawing forth â€”the abstraction & imagenation if one willâ€” of a new specific thing out from the general or a more general â€”which is to say the. Continue Reading. The highest level is no abstraction at all, but the completion, The Number of numbers, Form of forms, Thing of which things are abstracted : Totalus, Existence itself, The Perfect Sphere of Infinite Dimension. *â€¢ This is an excellent question, by the way, & is one sadly almost never addressed properly in the established citadels of our field today â€¢...Â An abstract can also be a model of a thing, for instance an abstract of my academic paper is a brief, or more limited, model of my paper as a whole. Thinkers relevant to abstraction in mathematics. I am not a historian, but think that a serious and thorough history of 'abstract method', even only insofar as mathematics is concerned (let alone architecture or fine arts) would have to treat at least the following (there are at least three glaring omissions, which I neither have time nor knowledge enough to correct; I draw a line where I perceive.Â which is an advanced meta-mathematical research article on questions which can be seen to be relevant to the "abstract method". Moreover, in an unsual, yet pertinent, allusion to the non-mathematical concept of 'behaviorism', Alain ProutÃ© in his recommendable 400plus-page lecture notes A. ProutÃ©: Introduction Ã la Logique CatÃ©gorique. Chapter 1. Babylonian mathematics 1. A mathematical tablet 2. Tally of pigs 3. The â€˜stone-weighingâ€™ tablet YBC4652 4. Cuneiform numbers from 1 to 60 5. How larger cuneiform numbers are formed 6. The â€˜square root of 2â€™ tablet 7. Ur III tablet (harvests from Lagash). Chapter 2. Greeks and â€˜originsâ€™ 1. The Meno argument 2. Diagram for Euclid I.35 3. The ve regular solids 4. Construction of a regular pentagon 5. The â€˜extreme and mean sectionâ€™ construction 6. How to prove â€˜Thalesâ€™ theoremâ€™.Â Their audience will rarely be students of history; although they are no longer conned to battles and sieges, the origins of the calculus are still too hard for them. Students of mathematics, by contrast, may nd that a little history will serve them as light relief from the rigours of algebra. The first is that mathematical abstraction can play an important role in shaping the way we think about and hence.Â On the origin of species by means of natural selection, or the preservation of favoured races in the struggle for life. London: Murray. Facsimile, Cambridge: Massachusetts: Harvard Press, 1964. Edwards, A. W. F. (1994). The fundamental theorem of natural selection. Biological Review, 69, 443â€“474. Article Google Scholar.